Prediction of Oblique Perforation of Concrete Target by Hard Projectile

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Introduction

Oblique perforation of concrete target by a rigid projectile is investigated through a mechanical model, which characterizes the physical phenomena into three general stages, namely initial cratering, tunnelling and shear plugging. It is based on the theory of dynamic cavity expansion and plug formation. Equations for residual velocity and directional change are derived. Predictions on ballistic perforations of thin and thick targets correlate well with experimental data.

Dynamic cavity expansion theory

In the process of oblique penetration through a concrete target, the initial crater is assumed to be of an oblique-crossed conical shape having an axial depth $k_d$. The axial resistant forces on the projectile nose during the cratering and penetration processes are respectively,

$$ F = \begin{cases} c \frac{x}{d} & \text{for } \frac{x}{d} \leq k, \\ \frac{\pi d^2}{4} \left( S_f + N^* \rho V^2 \right) & \text{for } \frac{x}{d} \geq k \end{cases} $$

where $x$ denotes the instantaneous penetration depth, $\rho$ is the density of concrete and $S$ is an empirical constant related to the unconfined compressive strength of concrete $f_c$. Denoting $I$ as an impact function, $N$ as a geometry function, and $N^*$ as a nose factor, we have

$$ I = \frac{Mv^2}{d^3 S_f}, \quad N = \frac{M}{\rho d^3 N^*} \quad \text{and} $$

$$ N^* = -\frac{8}{d^2} \int_0^H \frac{\gamma y/3}{1 + \gamma^2} dx. $$

where $\gamma$ is the geometric function of projectile’s nose curve. Previous studies showed that $I$ and $N$ are the two dominating factors in describing the penetration process.

Directional change within the initial cratering stage

By evaluating the kinetic energy consumption normal to the emerging path, the angle of directional change $\delta$ can be written as

$$ \sin^2 \delta = \frac{\delta}{1 + \frac{1}{N}} \left( \frac{1}{I + \frac{1}{N}} \right)^2 $$

for $\frac{x}{d} \leq k$;
\[
\sin^2 \delta = \frac{k \pi \delta \sin \beta \left( \frac{1}{I} + \frac{1}{N} \right)}{4}, \text{ for } \frac{x}{d} < k; \\
\text{(3b)}
\]

in which \( I_e = \frac{MV_e^2}{d^3 Sf_c} \). \( V_e \) denotes the velocity of projectile at transition from initial cratering to shear plugging for thin target. But for thick target, \( V_e \) denotes the velocity at the end of tunnelling process. Equations 3a and 3b quantify the effect of geometry of projectile, material of target, impact velocity and initial obliquity, etc.

### Shear plugging

In oblique impact, the plug (rear crater) is idealized as an oblique-crossed conical crater having a cone slope angle \( \alpha \) and an oblique-crossed angle \( \beta \pm \delta \) (see Fig. 3). The shear surface area of the oblique-crossed conical plug \( A_s \) and the residual thickness \( H^* \) are functions of \( \alpha, \beta \) and \( \delta \). The functions are kinematic relations.

The failure stress in pure shear is defined as \( \tau_f = f_c / \sqrt{3} \). Since concrete is a brittle material, the shear plug is assumed to be separated from its parent soon after shear failure occurs. In fact, the shear plug often flies off in fragments. Nevertheless, simple assumption is made that the conical surface is the only failure surface. Therefore, the axial resisting force (in Equation 1) to the projectile is equal to the total shear force’s component along the direction of motion, i.e.

\[
F = f_c A_s \cos \alpha / \sqrt{3} \\
\text{(4)}
\]

Equation 4 manifests the dependence of \( H^*/H \) on the initial velocity \( V_0 \) and the geometric configuration in both cases of \( x/d \leq k \) and \( x/d > k \).

### Ballistic performance

The ballistic limit is obtained when \( V_e = I_e = 0 \). We denote \( V_{BL} \) as the ballistic limit and its corresponding impact function as \( I_{BL} = \frac{MV_{BL}^2}{d^3 Sf_c} \), and \( I_{BL}, c_{BL}, A_{BL}, H_{BL} \) as the corresponding values of \( \delta, c, A, H \) at the ballistic limit respectively. The ballistic limit for a concrete target impacted by a hard projectile can be written as

\[
V_{BL} = \sqrt{\frac{d^3 Sf_c}{M} \cdot I_{BL} }, \text{ and } \\
I_{BL} = \frac{2c_{BL}}{f_c A_{BL} \cos \alpha} \sqrt{\frac{3k}{\pi} \left( \frac{k \pi}{4N} \right)^2 - \frac{1}{N}} \\
\text{for } \frac{x}{d} < k, \text{ and } \\
I_{BL} = N \sec^2 \delta_{BL} \left( 1 + \frac{k \pi}{4N} \right) \times \exp \left[ \frac{\pi \sec (\beta + \delta_{BL})}{2N} \left( 1 - \frac{H_{BL}^*}{H} \right) - \frac{k \pi}{2N} \right] - 1 \\
\text{for } \frac{x}{d} > k. \text{ (5c)}
\]

Thus when \( V_0 > V_{BL} \), the projectile perforates the target with a residual velocity as follows.

\[
V_e = \sqrt{\frac{d^3 Sf_c}{M} \cdot I_e }, \text{ and } \\
I_e = \left[ \frac{1}{1 + \frac{2c}{f_c A_s \cos \alpha} \left( \frac{3k}{\pi} \left( 1 + \frac{k \pi}{4N} \right) \right)} \right]^{-2} \left( \frac{1}{N} \right)^{-\frac{1}{N}} \\
\text{for } \frac{x}{d} < k, \text{ and } \\
I_e = -N + \frac{1}{1 + \frac{k \pi}{4N}} \exp \left[ \frac{\pi \sec (\beta + \delta)}{2N} \left( 1 - \frac{H_{BL}^*}{H} \right) - \frac{k \pi}{2N} \right] \\
\text{for } \frac{x}{d} > k. \text{ (6c)}
\]
Figure 5. Ballistic performance of thin target under 30°-oblique impact by hard projectile [test result by Buzaud et al (1999)]

Note that for common cases such as normal impact or $N \gg 1$, much simpler formulae can be deduced.

The above-mentioned formulae are used to predict the residual velocity of projectile and the angular directional change, and compare with available experimental data. Close agreements are observed in both thin and thick targets (see Figures 4-6).

Axial Flexibility of Completely Overlapped Tubular K(N) Joints

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Introduction

Offshore structures are commonly analysed with the structural members modelled as one-dimensional beam elements with rigid joint connections. It is assumed that the reactions at the brace ends have no effect on the chord wall. However, the chord wall actually deforms locally under the brace loads. This effect, which is generally referred as the local joint flexibility (LJF), could have significant influence on the deflections, buckling loads and distributions of nominal stresses in both elastic and elastic-plastic behaviours of structures.

Presently, there are limited parametric equations to determine the LJF for both uni-planar and multi-planar gap joints. However, only the simple gap joints rather than the stiffened joints are well studied and they have shown considerably high degree of flexibility. For the completely overlapped joints, no relevant data is currently available for references. The only one, which is related to the current research in some ways, is the study of the 100% overlapped flexible joints on the behaviour of rectangular hollow section trusses by Coutie and Saidani [1].

In this current study, the influences of various diameters and wall thicknesses of the chord and the through brace on the LJF of the joints are investigated based on the linear elastic behaviour. Gap sizes between the chord and the lap brace on the through brace surface are also investigated.

Finite element analysis

The finite element (FE) model has been verified and calibrated against the results from the experiment (Figure 1) [2]. The FE package MARC [3] is used to perform the analysis. The definition of the geometric parameters of the completely overlapped joint is given in Figure 2. The two ends of the chord and the end of the through brace are pinned supported except the end of the lap brace, which is free to move in all directions. The chord and the through brace lengths are 6 times while the lap brace is 3 times their respective diameters. Only axial compression is considered in this study. The weld elements are excluded in the FE
Determinant of local joint flexibility

The local deformation at the joint intersections is obtained directly in the direction normal to the chord (or through brace) axis from the chord (or through brace) ovality without considering the beam-bending movement and is referenced to the work of Fessler et al [4] (Figure 3). The flexibility coefficient \( \beta_{CT} \) of the joint subjected to brace end axial loading is obtained from the following formula.

\[
\beta_{CT} = \frac{d_T}{d_T} \quad \gamma_{CT} = \frac{d_T}{d_T} \\
\theta_{CT} = \frac{d_T}{d_T} \quad \tau_{CT} = \frac{d_T}{d_T}
\]

Where,

- \( E \) = Young’s modulus
- \( D \) = Diameter of chord (or through brace)
- \( P \) = Brace end axial load

\[
\delta = \left( \delta_1 - \delta_1 \right) + \left( \delta_2 - \delta_2 \right) + \left( \delta_3 - \delta_3 \right) + \left( \delta_4 - \delta_4 \right)
\]

Effect of chord diameter, \( D \)

As shown in Figure 4, \( \beta_{CT} \) almost constant as \( \beta_{CT} \) increased from small to medium values. Although \( \beta_{CT} \) of the joint increased with the gap, the difference of deformations was found small. Thus, the chord diameter showed insignificant effect on the LJF.

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Figure 2. Definition of parameters of the completely overlapped tubular joint

Figure 3. Determination of local joint flexibility

Figure 4. Effect of chord diameter, \( D \)
Effect of chord wall thickness, $T$

As illustrated in Figure 5, the maximum difference ($f_{\text{coeff}} = 24.6$) of those two curves occurred at the gap of $1.0d_T$. The chord wall thickness showed little effects on the LJJ.

Effect of through brace diameter, $d_T$

For all the curves, $f_{\text{coeff}}$ increased with the gap size (Figure 6). At the gap length of $2.5d_T$, $f_{\text{coeff}}$ for $\beta_{TL} = 0.331$ was almost 13 times higher than that for $\beta_{TL} = 0.768$. From the minimum gap size (= 51 mm) to the gap of $2.5d_T$, the increments of $f_{\text{coeff}}$ about 7.6 and 5.2 times for $\beta_{TL} = 0.331$ and $\beta_{TL} = 0.768$ respectively.

Effect of through brace wall thickness, $t_T$

Two different wall thickness ratios were studied: $\tau_{TL} = 0.747$ and $0.28$ with the corresponding $\tau_{CT} = 0.375$ and $1.0$ (Figure 7). As the wall thickness increased, $f_{\text{coeff}}$ of the joint reduced. The reduction was found to be drastic at a large gap. $f_{\text{coeff}}$ reduced by 6.16 times at a gap of $2.5d_T$. The joint with the thicker wall ($\tau_{TL} = 0.28$) had a less significant effect on the LJJ of the joint.

Conclusion

The effects of chord geometric parameters on the LJJ of the completely overlapped joints are insignificant but the effects of through brace geometric parameters are most crucial. A higher LJJ is obtained at a large $\beta_{CT}$ and $\gamma_T$ with small $\beta_{TL}$. The effects of gap sizes between the chord and the lap brace on the through brace surface show that only at a large gap, the LJJFs of the completely overlapped joints may be close to that of simple T/Y-joints. Thus, considering the completely overlapped joint as two separate T/Y-joints is inappropriate as the LJJ can be significantly overestimated which can lead to an error in the global analysis of structures.

References


Synthetic Seismograms of Distant Sumatran Earthquakes

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Introduction

Singapore is located in a low-seismicity region, where the active seismic sources are located more than 300 km away, along and off the western coast of Sumatra. It is natural that records of seismic ground motions in such a low-seismicity area are scarce. To access the regional seismic hazard thoroughly, the limited number of recorded data should be augmented by synthetic seismograms. In this paper, ground motion simulations of two distant Sumatran earthquakes recorded recently in Singapore are presented to justify the use of synthetic seismograms in the regional seismic hazard assessment.

Network of seismic stations

In accordance with the increasing public awareness on seismic hazard in Singapore, the Meteorological Service Singapore (MSS) established a national network of seismic stations in September 1996, to gather information on the regional seismic activities. The network is composed of one broadband Global Seismograph Network (GSN) station, four teleseismic stations and two borehole arrays. The GSN station is the only station located on a rock-outcrop site. The station, situated at the centre of the Singapore island, is equipped with a comprehensive set of sensors to record ground tremors continuously.

From the establishment of the seismic array to December 2001, 112 earthquakes with $M_w \geq 5.0$ have occurred in Sumatra. Ninety-eight events occurred on the plate boundary or within the subducting plate, and the remaining 14 events were shallow strike-slip events along the Sumatran fault. Out of the 112 earthquakes, six events have generated ground tremors that caused perceptible levels of vibrations to residents of high-rise buildings in Singapore. Ground tremors from 37 subduction earthquakes and seven Sumatran-fault events have been recorded by the GSN station. The majority of the records are very weak, with poor signal-to-noise (S/N) ratios and PGAs not greater than 0.1 mm/s². Out of the seven recorded Sumatran-fault events, only two records show good S/N ratio and have recorded PGVs greater than 0.2 mm/s. The epicentres of the two events are shown in Figure 1.

Recorded ground motions

The recorded ground velocities of the two events are shown in the uppermost panels of Figures 2 and 3, respectively, where the first and second columns show the NS and EW components, and the last column shows the UD component. The time 0 s refers to the rupture initiation time. The records have been baseline corrected and low-pass filtered at 2.0 Hz to remove the high-frequency components due to local geology and environmental noises. The number at the top left corner of each panel indicates the PGV in cm/s. The acceleration response spectra of 5% damping ratio are depicted by the solid lines in the bottommost panels of each figure.

Simulation of recorded data

The ground motion simulations used in this study follow the reflectivity algorithm developed by Kohketsu [1]. A typical seismic source is modelled as a single dislocation point source, whose source time function is approximated by a ramp function.
The rise time of the function is determined from stress drop. The crustal structure is modelled as a horizontally layered medium. The regional crustal structure of Sumatra is extracted from the global crustal model CRUST 2.0, which is a recent 2° × 2° global model for the Earth’s crust based on seismic refraction data published in the period of 1948 – 1995. The appropriateness of the source and path modelling plays an important role in obtaining a reliable synthetic seismogram.

The ground motions of the recorded events are simulated. The panels in the second row of Figure 2 show the simulated ground velocities of the 19961010 event in NS, EW and UD directions. The source rise time is calculated to be 3.6 s, assuming a stress-drop of 100 bars. The upper cut-off frequency of the simulation is 2.0 Hz. The simulated waveforms appear to agree substantially well with the recorded data, except that the duration of recorded ground motions in the horizontal directions appears to be longer than that of the simulated ones. The acceleration response spectra (5% damping ratio) of the simulated and recorded ground motions are compared in the bottommost panels of the figure, which show good agreement between them.

Figure 3 shows the recorded and simulated ground velocity time histories and acceleration response spectra (5% damping) of the 19970820 event. The simulated response spectra agree well with those of the recorded data within the natural periods from 0.5 to 20 s. The recorded ground velocities seem to have a longer duration than the simulated ones, which might be due to the scattering caused by small-scale heterogeneities that cannot be simulated by the one-dimensional layered model.

Conclusion

The good agreement between the recorded and simulated ground motions indicates that the regional crustal structure extracted from the CRUST 2.0 represents well the actual crustal structure. It also implies that it is possible to derive representative ground motion relationships for the region based on the method and crustal structure used in the simulations.

Reference


Optimal Vibration Control of Smart Structures

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Introduction

Vibration control of flexible structures has been a major research topic over the past few decades. The smart structure, which contains the main structure and the distributed piezoelectric sensors and actuators (S/As), as well as the control system, can simultaneously detect certain vibration modes and generate control forces to reduce the vibration of the structure. The piezoelectric sensors and actuators have to be of suitable size and be appropriately located to ensure maximum effectiveness for vibration control. As can be found from the available literatures, most attention has been paid to the geometric optimization of S/As such as their placement, size as well as thickness, but integrated control system optimization considering the geometric distribution of the piezoelectric patches and the feedback control gains of the control system was rarely investigated.

In this study, the parameters of vibration control system, including the geometric distribution of piezoelectric sensors and actuators bonded on smart structures and the feedback control gains of the actuators, have been optimized simultaneously for vibration suppression of the structures. The energy dissipation method has been employed as the optimization criterion. A real-encoded genetic algorithm (GA) has been developed and applied to search for the optimal placement and size of the piezoelectric patches as well as the optimal feedback control gains. The results showed that the control effect could be significantly enhanced with appropriate distribution of piezoelectric patches and feedback control gains.

Modelling of Sensing and Actuating

Consider a cantilever beam, as shown in Fig. 1, bonded with
piezoelectric actuators on the upper surface and sensors on the lower surface oppositely. Assume that $m$ pieces of collocated sensors and actuators are bonded on the beam. The piezoelectric material is supposed to be transverse isotropic and polarized in the $z$ direction.

Considering one piece of piezoelectric patch exerted with a voltage $V_a$, Bailey and Hubbard (1985) deduced the following equation.

$$E_{ij} \frac{\partial^2 w}{\partial x^2} + \rho A \frac{\partial \dot{w}}{\partial t} = M [\delta(x-x_i) - \delta'(x-x_i)]$$

where $w$ is the deflection of the beam. Introducing state vector $X = [\eta_1, \eta_2, \ldots, \eta_n, \eta_{n+1}, \ldots, \eta_N]^T$, where $\eta_i$ and $\eta_j$ ($i = 1, 2, \ldots, n$) are the modal amplitude and its derivative with respect to time, respectively, using the modal decomposition method and considering $m$ pieces of collocated sensors and actuators, the above equation can be transformed into

$$\dot{X} = A X + B V$$

where $B$ is the output voltages of the sensors.

More details of modelling of sensing and actuating can be referred to Jin et al (2001).

**Control method and optimization criterion**

The most attractive methodology that accounts for transient vibration responses is characterized by the maximization of the dissipation energy extracted by the feedback control system. The constant negative velocity feedback method is adopted as $V_a = -G \dot{X}$, where $G$ is the feedback gain matrix. The corresponding closed-loop state-space equation is $\dot{X} = AX + BV$.

The more the energy dissipated by the control system, the less the energy is stored in the system. The integrated total energy stored in the system can be written as $W = -X^T(t_0)P\dot{X}(t_0)$, where $X(t_0)$ is the initial state of the vibration system and $P$ is the solution of the Lyapunov equation $A^T P + PA = Q$. Thus, the problem can be expressed as a nonlinear optimization problem with constraints as

$$\min_{x_i, x_j, G, m} J(x_i, x_j, G, m) = -X^T(t_0)P\dot{X}(t_0) \quad (i = 1, 2, \ldots, m)$$

subject to $x_i \in X_i, x_j \in X_j, G \in G'$, where $X_i, X_j$ are the boundaries of the placement of the piezoelectric patches and the maximum feedback control gain matrix.

**General formulation for real-encoded GAs**

GAs have recently been recognized as a promising tool for numerical optimization of structural design problems. The fundamental mechanisms leading the GA search process are the equivalents of natural selection, crossover and mutation. Michalewicz (1994) has compared real-encoded and binary GA and shows that the former is more efficient in terms of CPU time. In this study, the uniform mutation and the arithmetic crossover are adopted. Let $a_i$ and $b_i$ be the lower and upper bound, respectively, for each variable $i$. Uniform mutation sets the new variable value equals to $U(a_i, b_i)$, which is a random number uniformly distributed between $a_i$ and $b_i$. Real-encoded arithmetic crossover produces two complimentary linear combinations of the parents, $r = U(0,1)$.

Since GAs can be directly used for unconstrained problems only, our optimal design problem needs to be transformed into an unconstrained problem by introducing the exterior penalty functions. Mathematically, the evaluation of the objective function can be represented in the following form:

$$\phi(X, G) = J(X, G) + \sum_{i=1}^{m} \left( \sum_{j=1}^{N} r \left[ \min\{0, (x_{i,j} - x_{i,j})\} \right]^2 \right)$$

In this study, the initial population is randomly created and the population size is set as 60. The crossover probability $P_c$ is set as 0.87, and the mutation probability $P_m$ is set as 0.01. The stopping criterion $G_{max}$ is the preset maximum number of generations. Usually after this generation, there is no more improvement in the optimum solution. In this study, $G_{max}$ is preset at 200.

**Illustrative example**

A simple cantilever beam which has been studied by Lee and Chen (1994) is investigated. The characteristic data of the beam are listed in Table 1. In the following design, the first four vibration modes are considered to be the controlled modes. The initial conditions of the generalized coordinate vector are given by:

$$\eta(0)^T = [0 \ 0 \ 0 \ 0], \quad \eta(0)^T = [0.525 \ 0.292 \ 0.171 \ 0.122].$$

In this case, besides the geometric constraints, a simple bound of the feedback control gain matrix $G$, that is, $0 < G_{ii} \leq 50$. Three cases with one to three pieces of piezoelectric patch have been considered. The optimization results are shown in Table 2. The time behaviours of the vibration modes without control and with control using one to three pieces of piezoelectric patch are shown in Figs. 2 to 3. From the Table and Figures, it

| Characteristic data of a cantilever beam and the piezoelectric patches |
|-----------------------------|-----------------------------|
| Mass/length                | $m = 1.496 kg / m$          |
| Length                      | $L = 3.81 m$                |
| Modulus of elasticity      | $E_s = 2.07 \times 10^4 N/mm^2$ |
|                            | $E_p = 6.3 \times 10^4 N/mm^2$ |
| Width                       | $b = 0.01 m$                |
| Height                      | $t_b = 0.002 m$, $t_p = 2 \times 10^4 m$ |
| Piezoelectric constant     | $d_{31} = 1.2 \times 10^{-10} mN/V$ |
| Capacitance                 | $C_p = 6.287 \times 10^7 pC$  |
| Damping coefficient         | $\zeta = 0.01$              |
Conclusion

In this article, the integrated optimization design of the vibration control system including the placement and size of the piezoelectric patches as well as the feedback control gains has been formulated. A real-encoded GA has been developed and applied to this optimization problem. The energy dissipation method has been adopted for the vibration suppression of the structure. The results of a cantilever beam model show that using this integrated optimization of geometric distribution of the piezoelectric patches and the feedback control gains, the vibration of the structure can be effectively suppressed.

References

Finite Element Mesh Generator and Analysis of Cracked Tubular K-Joints

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Introduction

Tubular K-joints configuration as shown in Figure 1 has been widely encountered in the offshore structures. As a result, it is important to study the fatigue, fracture and the crack growth behavior of these K-joints so as to predict its remaining service life. The prediction of the cracked tubular K-joints depends very much on the accuracy of the stress intensity factors. Up to now, no published stress intensity factors results for K-joints have been published. This is mainly due to the fact that modelling the crack surface together with the weld profile accurately is a very complex undertaking.

Mesh generation of a surface crack

A new mesh generation method is proposed in this study whereby the K-joint model is divided into different zones as shown in Figure 1. Each of these zones consists of a different density of elements. In the far field (Zones A, E and H), only one layer of 3D elements is employed since the stress distribution along the thickness direction is nearly uniform. Fine mesh with more than one layer of elements are generated near the intersection (Zone CF) so that the FE model can capture the stress concentration near the intersection. Between the refined regions and the far field regions, transitions regions (Zones B, D & G1) are used to increase the number of layers of elements in the thickness direction from one to three. The mesh of each zone is generated individually and then merged together to form the complete mesh.

When generating the FE mesh with a surface crack, part of the sub-zone called CRBLOCK in Zone CF is extracted and crack elements are generated separately for the block of elements as shown in Figure 2. After this block of elements is generated, it is then merged to the mesh in Zone CF. It is noted that the number and the location of elements extracted from Zone CF depend on the length and the location of the crack at the welded joint.

Figure 3 shows the FE mesh generated near the crack tip for a surface crack. Around the crack tip, two rings of refined elements are employed around the semi-elliptical surface crack. The first ring consists of 27 quarter-point quadratic prism elements along the thickness. For the second ring, the mesh density is reduced to 9 prism elements. After the crack elements are generated, the whole tube of the elements is inserted into the block extracted from Zone CF. Finally, the block is merged back to Zone CF to form the complete FE mesh.

Figure 1. Mesh of K-joint with different zones
Stress intensity factors

In practice, the two most common methods to evaluate the stress intensity factors are the J-integral and the displacement extrapolation. J-integral is a measure of the strain energy in the region of the crack tip, and relationship between the J-integral and the stress intensity factors is as follow:

\[
J = \frac{1}{8\pi} K^T B K \quad (1)
\]

where \( K = [K_I, K_{II}, K_{III}]^T \), and \( B \) is called the pre-logarithmic energy factor matrix.

However, it is not feasible to obtain \( K_I \), \( K_{II} \), and \( K_{III} \) from \( J \) directly. This means the J-integral method cannot be easily used to analyze the mixed mode problems directly. Instead, an interaction integral method is used to calculate \( K \) from the J-integral as follow:

\[
K = 4\pi B \cdot J_{int} \quad (2)
\]

where \( J_{int} \) is the interaction integral. The latest version of ABAQUS software can obtain \( K_I \), \( K_{II} \), and \( K_{III} \) and J-integral by above interaction integral method directly.

Displacement extrapolation is another effective method to calculate the stress intensity factors. The equations used to calculate the stress intensity factors based on this method can be expressed as follow:

\[
K_I = \frac{G}{2(1-v)} \sqrt{\frac{2\pi}{r}} v, \quad K_{II} = \frac{G}{2(1-v)} \sqrt{\frac{2\pi}{r}} u, \quad K_{III} = G \sqrt{\frac{2\pi}{r}} w \quad (3)
\]

where \( u, v, w \) denote the local radial, normal and tangential displacements of the nodes on the crack surface respectively, \( G \) is the shear modulus and \( v \) is the Poisson’s ratio. During the analysis of SIFs for surface crack in tubular joints, the three important locations are the deepest point of the crack and the two crack tips.

Numerical results

A K-joint with 1610mm brace length and 2100mm chord length is subjected under balanced axial loadings. The gap is 40mm. A semi-elliptical surface crack with \( a=10\text{mm} \) and \( c=60\text{mm} \) is modeled at the weld toe around the peak stress location. ABAQUS was then used to analyze the K-joints and to produce the stress intensity factors. Figure 4 shows the plotted stress intensity factor values against \( \phi/\pi \) using the displacement extrapolation method and the J-integral method.

Conclusion

A new mesh generation method of K-joints with a surface crack is proposed in this study. Hexahedral, tetrahedral, pyramid, prism and quarter-point crack tip (collapsed prism) elements are used to model the surface crack, and this enables a high quality mesh to be produced especially around the crack front. It is obvious that both displacement extrapolation method and J-integral method can be used to obtain the stress intensity factors of any K-joints. From the results shown in Figure 4, the stress intensity factors obtained from the two methods are almost the same, except the values at the crack tips. The stress intensity factors at the crack tips obtained from J-integral have higher values than the results obtained from displacement extrapolation method. Usually the stress intensity factors obtained from the J-integral are generally more accurate than the results obtained from the displacement extrapolation method at the crack tips.
Structural Assessment of Highway Bridge Upgrading

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Introduction

The Land Transport Authority (LTA) of Singapore has a continuing program of highway bridge upgrading, to refurbish and strengthen bridges in order to allow for increasing vehicle traffic and axle loads. One subject of this program has been Pioneer Bridge: a 30 year old structure near Jurong Port having a rather short span of 18.8m. The bridge comprises 37 pre-cast pre-tensioned inverted T-beams in the traffic direction and these are tied together by 25 equi-spaced cast in-situ transverse diaphragms. T-beams carry a 150-300mm thick deck slab.

The bridge was assessed analytically by LTA to evaluate its strength and to identify any defects and deficiencies in the structure. Strengthening works were proposed in which the simply supported one-way span would be converted to a jointless structure with rotational restraint at the abutments, to eliminate bearings and enhance load distribution at supports.

Experiment-based structural assessments of the bridge were conducted before and after upgrading works including strengthening. Each assessment exercise comprised two components: a full-scale dynamic test carried out in a single day without closing the bridge and a finite element model updating exercise to identify structural parameters and understand the bridge’s dynamics. Parallel to the dynamic testing and updating, live strain monitoring exercises were undertaken before and after this assessment.

Dynamic testing and system identification

For producing measurable dynamic response of the bridge, hammer testing and traffic excitation were evaluated and found to be less viable for this particular bridge than forced vibration using a portable (but very heavy) long stroke electrodynamic shaker. Over 80 measurement points were used in order to cover the two-dimensional structure with sufficient spatial resolution to identify the set of two-dimensional vibration modes whose shapes would depend on the stiffness and mass distribution in the bridge. Frequency response functions were obtained using the same excitation signals, a broadband ‘chirp’ repeated for different arrangements of the thirteen accelerometers deployed. The 4-32Hz frequency range for bridge modes was identified by studying how the structure responds to light vehicles.

As the bridge could not be closed to traffic, the testing was done on the quietest day of the week, Sunday. Even so, usable measurements could only be done when the bridge was free of traffic. Using time series for each set of accelerometers, cross-power spectral matrices were created and merged through normalisation with respect to a common channel. A ‘response only’ system identification procedure (ERA) was then used to extract the mode frequencies, damping ratios and shapes from the cross-powers, while conventional circle-fitting to frequency response functions (FRFs) was used to obtain the modal masses. In order to manage the data processing, system identification, presentation of results and comparison with finite element predictions a MATLAB-based code MODAL was developed.

Figure 2 shows correlation between the second vibration mode of the prototype upgraded bridge (circles) and a finite element model (solid lines).

Figure 1. Pioneer Bridge

Figure 2. Second mode of vibration: experimental (circles) and updated analytical (lines)
Figure 3. Driving point inertance FRF before (above) and after (below) upgrading. Units are (1000kg)^{-1}

The updating procedure applied here used the sensitivity matrix approach.

These investigations showed that bridge stiffness increased considerably due to the upgrading, but that full end restraint had not been achieved. The first natural frequency increased from 5.55 Hz to 8.31 Hz, but simulation with the validated FE model showed that with the T-beam ends fully fixed a fundamental natural frequency as high as 12.21 Hz could have been achieved.

Live strain measurements

The bridge monitoring program involved measurement of dynamic strain due to traffic at the bridge’s mid-span using a purpose made bridge monitoring system consisting of four demountable strain gauges. The objective of the bridge monitoring exercise was to assess the level of live loading on the bridge, to determine the daily loading patterns, to predict the level of ultimate live loading, to assess the bridge’s structural response before and after upgrading works, to compare impact factors before and after upgrading works and finally to estimate the load carrying capacity before and after upgrading works. Figure 4 shows a typical dynamic strain record.

The ultimate live load was judged to be the most severe combination of 120 year values of average strains recorded by each of the four strain gauges. 120 year strains were estimated from daily maximum strains using the Gumbel distribution (Figure 5). The ultimate live load strain was then combined with dead load strain estimated from an updated bridge model in order to estimate the load carrying capacity of the bridge. The ratio of assessment load effect i.e. ultimate live load plus dead loads to the assessment resistance i.e. yield load of pre-stressing tendons, was found to be approximately 0.8 and 0.5 before and after upgrading respectively confirming the effectiveness of the upgrading exercise. This practical approach to bridge assessment takes advantage of bridge specific live loads and a representative structural model obtained via modal analysis and model updating.

Finite element model updating

Using frequency and mode shape information, it is possible to find an optimal match between a finite element model and experiment by adjusting the ‘certainly uncertain’ parameters such as bearing rotational stiffness. Having obtained a validated model, it should be possible to provide a far more reliable assessment of load capacity than by using a relatively simple analytical (beam) model. The updating procedure applied here used the sensitivity matrix approach.

Conclusion

A procedure has been developed that uses modal testing to provide a validated analytical model of a bridge, which can then be used with live load monitoring to predict or assess the effectiveness of upgrading. Depending on bridge size, different test procedures would be used.
Numerical Simulation of Blast Wave Propagation in Soil Mass

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Introduction

The blast wave propagation in soil mass is an important topic in mining, construction and defence engineering; however, it is very difficult to model due to the complex behaviour of the soil mass. The current practice in predicting the blast wave propagation in soils is mainly based on empirical formulae obtained from field blast tests. Although there also exist many numerical models on the behaviour of soil mass [1], these models are suitable only for static or transient loading. Modelling the soil response under high amplitude blast loading requires more sophisticated description of the soil behaviour. In this project a three-phase soil model for shock loading is developed and applied to simulate the blast wave propagation in soil mass. The soil is modelled as a three-phase mass that consists of solid particles, water and air. These solid particles form a skeleton and their voids are filled with water and air. The water and air deform with the skeleton. The model is implemented into a commercial hydrodynamic software AUTODYN as its user subroutines.

Formulation of numerical model

Soils have a complex structure, usually it consists of mineral particles, water and air. Primarily, two deformation mechanisms exist in soil [2]: I. the elastic deformations of bonds on the contact surfaces of particles at low pressure and a failure in bond and displacements of the particles at high pressures. II. the deformation of all the soil phases, determined by their volume compression.

Figure 1 describes the soil deformation under shock loading. A, B, C correspond to the deformation of the solid particles, water and air respectively. D is the friction between the solid particles, and E is the elastobrittle bond linkage between the solid particles. The first deformation mechanism corresponds to the elements D and E while the second deformation mechanism corresponds to the elements A, B and C.

Under rapid shock loading there is not enough time to squeeze the gas and water from the soil skeleton. The overall equilibrium equation can be written as

\[ \sigma_{ij} + \rho \dot{u}_i = \rho u_i \]  

where \( \sigma \) is the total stress, \( u \) is the displacements (average) of the solid matrix, \( g \) is the vector of gravitational accelerations, \( \rho \) is the mass density and \( \dot{u}_i = \frac{\partial}{\partial t} u_i \).

The equation of state of the soil under shock loading is

\[ \rho \frac{\partial p}{\partial t} - \left( \frac{\partial V_r}{\partial p} \right)^{-1} \left( \frac{\partial \sigma_r}{\partial V_r} \right) \frac{\partial V_r}{\partial p} = 0 \]  

where \( p \) is the total pressure, \( p_a \) is the pressure borne by the friction between the solid particles, \( p_b \) is the pressure borne by the water and gas, \( p_c \) is the pressure borne by the bond between the solid particles, \( V \) is the volume of a soil element, \( V_w, V_g \) and \( V_p \) are the volume of water, air and soil particles phase. \( \frac{\partial V_r}{\partial p}, \frac{\partial \sigma_r}{\partial p} \) are given by their independent equation of state or stress-strain relation.

The stress and strain relationship for the soil skeleton [3]:

\[ d\sigma_{ij} = K d\varepsilon_{ij} + 2G d\varepsilon_{ij} \]  

where \( d \varepsilon \) is the deviatoric strain, \( K \) is the elastic bulk modulus, and \( G \) is the elastic shear modulus. \( \delta \) is a positive scalar factor, \( Q \) is the plastic potential function.

To account for strain rate effects, the modified Drucker-Prager yield function is adopted,

\[ f = \sqrt{J_2} - (\alpha I_1 - k)(1 + \beta \ln \varepsilon_{eff} \varepsilon_0) = 0 \]  

in which \( \varepsilon_0 \) is the reference effective strain rate, \( \alpha \) and \( k \) are material constants, \( \beta \) is the slope on the strength vs. logarithm of strain rate curve, \( \varepsilon_{eff} \) is the effective strain rate, \( I_1 \) is the first invariant of the stress tensor, and \( J_2 \) is the second invariant of the deviatoric stress tensor.

For soils, the non-associated flow rule is suitable,

\[ Q(\sqrt{J_2}) = \sqrt{J_2} - Y = 0 \]  

in which \( Y \) is the yield limit defined by the yield function.
Numerical simulation

The propagation of the blast wave in soils is studied numerically using the above described soil model. Figure 2 shows the configuration of the numerical setup. The relative volume ratio of each phase in soils is assumed to be 0.6:0.25:0.15 (solid particle: water: air). The weight of explosive (TNT) is 8kg. The JWL equation of state is adopted to describe the expansion of the explosion product of TNT. Figure 3 plots the time histories of the pressure and R-direction particle velocity at different locations. As shown, the blast wave has the feature of the shock wave in the near field; as the wave propagates, the waveform changes to the nature of a stress wave with reduced amplitude. The peak pressure and peak particle velocity attenuate with increasing distance from the charge.

Figure 4 shows a comparison between the numerically simulated and empirical attenuation relations of the peak pressure and peak particle velocity as a function of the scaled distance. The computed results generally agree well with empirical values. The computed attenuation trends appear to suggest a two-linear curve, which seems to be physically more reasonable because in the vicinity of the explosion centre the pressure is far larger than the yield stress of the soil skeleton; hence, all the soil phases deform and the second deformation mechanism dominates. With increasing distance, the strength of the soil skeleton tends to show more significant effect, thus the first deformation mechanism turns to govern. Further numerical investigation of these pertinent phenomena is being carried using the proposed three-phase model.

Conclusion

A new three-phase model is developed to describe the behaviour of soil mass under shock loading. Numerical simulation of the blast wave propagation in soil mass using this model produces reasonable results as compared to existing test data. Detailed numerical results project a much clearer picture of the associated phenomena and provide a deeper insight into the underlying mechanisms affecting the blast wave propagation in soil mass. The model can be used for extended numerical investigation.

References


### Design of a New Impact Striker Bar for Material Tests in a Split Hopkinson Pressure Bar

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#### Introduction

A large diameter split Hopkinson pressure bar (SHPB) is essential to test non-homogeneous brittle materials such as concrete, brickwork, rocks, and ceramics subjected to high rates of strain. To test such materials at a lower strain rate, however, demands more than just lowering the impact velocity of the striker bar. A new striker bar form must be designed to induce an incident stress wave with a long duration without abruptly increasing the incidence stress. Further, the stress magnitude must be sufficiently high to fracture the specimen and yet retain equilibrium of the stress/strain condition in the specimen. A finite difference scheme is used to predict the influence of arbitrarily-shaped striker bars on the incident loading waveform. A new impact striker generating a half-sine waveform is then designed and fabricated to test the hypothesis. Tests on 75mm diameter concrete specimens show good agreement with predicted response.

#### Development of striker bar

Consider the stress wave generated by impact of an arbitrary, non-uniform diameter striker bar of length \( L \) on a solid cylindrical input bar as shown in Figure 1. The striker bar has a variable diameter, which is a function of \( x \).

Analysis of the model is restricted to the striker bar because the solution for the wave motion along the input bar is established for a uni-directional elastic wave travelling along a constant diameter. It is assumed that the propagation of elastic waves is described by one-dimensional wave theory and the input bar is infinitely long. Thus, the governing differential equations, including boundary and initial conditions, for the problem may be written as:

\[
\frac{\partial}{\partial x} \left( A \frac{\partial u}{\partial t} \right) = c^2 \frac{\partial^2 u}{\partial t^2}
\]

\[
\frac{\partial u}{\partial x} = 0 \quad \text{at} \quad x = 0
\]

\[
E A \frac{\partial u}{\partial x} + \rho_2 c_2 \frac{\partial u}{\partial t} = -p c_1 A v_0 \quad \text{at} \quad x = L
\]

\[
u = 0, \quad \frac{\partial u}{\partial x} = 0 \quad \text{at} \quad t = 0
\]

where \( u \) is the displacement of the striker bar induced by the impact, \( A \) is the cross-sectional area as a function of \( x \), \( c \) and \( c_2 \) are the longitudinal wave velocity of the striker bar and input bar respectively, \( E \) and \( A_i \) are Young’s modulus and cross-sectional area of the striker bar at the impact end respectively, \( \rho_2 \) and \( A_2 \) are the density and cross-sectional area of the input bar respectively and \( v_0 \) is the impact speed.

The finite difference method is used to solve this problem, in which the length of the impact bar is divided into \( N \) equal portions by \((N+1)\) nodes. Using a central difference approximation and an explicit algorithm for time, the approximate displacement at each node is expressed as:

\[
u_{i+\Delta x} = \frac{c^2 \Delta x^2}{\Delta t^2} \left[ (A_{i+\Delta x} + A_{i-\Delta x}) u_i - (A_{i+\Delta x} - A_{i-\Delta x}) u_{i-1} \right] + 2u_{i-1} - u_{i-2} \quad \text{...}(2)
\]

\[
u_{i+\Delta x} = \frac{c^2 \Delta x^2}{\Delta t^2} \left[ (A_{i+\Delta x} + A_{i-\Delta x}) u_i - (A_{i+\Delta x} - A_{i-\Delta x}) u_{i+1} \right] + 2u_{i+1} - u_{i+2} \quad \text{...}(3)
\]

\[
u_{i+\Delta x} = \frac{c^2 \Delta x^2}{\alpha A_i \Delta x} \left[ (A_{i+\Delta x} + A_{i-\Delta x}) u_i - (A_{i+\Delta x} - A_{i-\Delta x}) u_{i+1} \right] + 2u_{i+1} - u_{i+2} \quad \text{...}(4)
\]

\[
\alpha = 1 + \frac{\rho_2 c_2^2 A_2 A_{i+\Delta x} \Delta t}{\Delta x A_i^2 E} \quad \text{...}(5)
\]

where \( u_i, \Delta t, t \), and \( u_{i+\Delta x}, \Delta x \) are displacements at node \( i \) and time \( t+\Delta t \), \( t \) and \( t+\Delta t \), \( \Delta t \) and \( \Delta x \) are the time step and mesh size respectively, and \( A_{i+\Delta x}, A_i, A_{i+\Delta x} \) are the cross-sectional area of the striker bar at half-mesh size before node \( i \), at node \( i \) and half-mesh size after node \( i \) respectively.

For a given impact velocity and shape of the striker bar, the displacement history can be obtained using eqns (2)-(5). The response of a number of striker bar forms is illustrated in Figure 2. In the computation, the velocity of the striker bar is assumed as 10m/s and the total length of the bar is 1 metre in all cases. The shape marked with an asterisk (*) was selected for fabrication and verification of
Figure 2. Incident stress waveform from various striker bar shapes

Figure 3. Dimension of proposed striker bar

Figure 4. Normalized response history of incident stress waveform

Comparison of experimental and analytical results

Measured incident stress history of three specimens and the predicted incident stress history derived from the proposed striker shape are shown in Figure 4. Comparison of the predicted and measured histories verifies the hypothesis, although the predicted result precedes the test data by a small time interval. This slight difference is attributed to the prediction process, in which several idealised assumptions were made and these had caused the predicted waveform to occur earlier than the measured histories of the three specimens. Nevertheless, the predicted curve attained the same stress magnitude as that obtained in the experiment.

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Enhancement of Beam-Column Joint by RC Jacketing

Introduction

In low to moderate seismicity regions, reinforced concrete buildings are designed to resist dead and live loads only. This results in joint cores with reinforcement detail that cannot fulfil seismic design criteria, thus making such building frames vulnerable to joint shear failure when subjected to lateral loading or ground excitation. Even in seismic regions, old buildings designed according to the then existing immature seismic design codes lack sufficient hoops inside the joint cores. The joint cores are the most critical components in such frames, and the ultimate failure of such frames under lateral loading would be due to the inadequate shear capacity of the joint core. Hence, such lightly reinforced joints need to be strengthened before exposing them to any form of lateral loading. Reinforced concrete (RC) jacketing is an effective method of retrofitting such connections. In this paper, the usefulness of RC jacketing technique to strengthen lightly reinforced beam-column joints is investigated experimentally.

Test details

As shown in Figure 1, a full-scale reinforced concrete sub-
assembly with a 3.7 m high column and a 5.4 m long beam is subjected to quasi-static reversed cyclic loading. Due to the set-up details, the effective height of the column and the effective length of the beam given by the distances between the centrelines of the supports at the two extremes are 3.2 m and 6.0 m, respectively. An axial compression equal to 10-15% of the section capacity is applied at the column-top, and equal and opposite displacement cycles are applied at the two beam-tips. The amplitude of the cyclic displacement is increased gradually to induce up to 3% radian storey-drift. Thereafter, the first phase loading is terminated, and the specimen is retrofitted. The retrofitted specimen is again subjected to gradually increasing cyclic displacements in a similar fashion until 5% radian storey-drift is induced. The geometrical dimensions and rebar details of the beam and column before and after retrofitting are shown in Figure 2. The 300×550 mm beam is reinforced with seven 32 mm diameter bars, five of them at the top and two at the bottom. Similarly, the 350×500 mm column in the original specimen is reinforced with eight 25 mm diameter bars. The stirrups in the beam comprise of four legs of 10 mm diameter bars spaced at 200 mm, and the ties in the column have two legs of 10 mm diameter bars with 150 mm spacing. The beam size is not changed after retrofitting, whereas the retrofitted column is 670×820 mm in cross-section. The RC jacket cast outside the central 2.7 m of the original column includes four 25 mm diameter longitudinal bars arranged symmetrically with 12 mm diameter ties spaced at 100 mm.

The details of the original specimen were taken from a typical RC building frame in a low seismicity region, and these details make the specimen of the undesirable strong-beam weak-column type. As the column ties of the original specimen are continued through the joint, the joint core has only three ties, which is not enough to satisfy seismic design requirements. Note that no additional stirrups are inserted in the joint core during retrofitting (see Figure 3b). Before casting the 160 mm thick RC jacket, the crushed concrete is cleaned and epoxy is injected into the small cracks in the joint panel. The compressive strength of the original concrete is 33.6 MPa whereas the concrete used in the RC jacket has a high compressive strength equal to 74.8 MPa. The yield strengths of the 32 mm, 25 mm, 12 mm and 10 mm diameter bars are 527 MPa, 527 MPa, 420 MPa and 356 MPa, respectively. LVDT transducers and load cells are used to measure the displacements and forces at the beam-tips and also at the column-top. In addition, a pair of diagonally arranged pi-gauges is used to monitor the shear deformation of the joint panel.

Results and discussions

A pair of orthogonal diagonal cracks in the joint panel of the original specimen was visible when the storey-drift reached 0.5% radian. During the higher displacement cycles, these diagonal cracks widened and some more hairline cracks emerged in the joint panel. Spalling of concrete from the joint panel started during the 1.5% radian storey-drift cycle. The loading of the original specimen was terminated after the joint panel was damaged severely (see Figure 3a) after the 3% radian storey-drift cycles had been applied. Note that the damage is mostly concentrated in the joint panel and very few cracks could be seen in the beam or the column. In the retrofitted specimen, cracks first emerged in the beam during the 0.5% radian storey-drift cycles. The column began to crack when the storey-drift reached 0.75% radian, and diagonal cracks emerged in the joint panel when the 1.0% radian storey-drift cycles were applied. At 1.75% radian storey-drift, concrete at the beam-column interface started crashing and an opening appeared at the corner of the interface, which widened during further loading cycles. On further loading, more cracks appeared throughout the specimen. The test was finally terminated after applying the 5% radian storey-drift cycles, when visibly significant damage occurred on the specimen (see Figure 3c). Significant damage could be observed in the beam and the column, and the diagonal cracks in the joint panel were not as wide as those in the original specimen.

Figure 4a shows the envelopes of the storey-shear force versus storey-drift loops recorded during the two loading phases. Here, the storey-shear force is the reading from the load cell at the column-top, and the storey-drift is the rotation of the line joining the beam-tips from the original beam axis; i.e. summation of the displacements of the two loading points.
divided by the effective beam length. A gradual reduction of the shear stiffness in the pre-peak region followed by a gradual degradation of the storey-shear force in the post-peak region can be noticed in both curves. As the weakest component of the original specimen was the joint and it was strengthened by RC jacketing, the shear capacity of the sub-assembly understandably increased after retrofitting. Surprisingly, the deformability of the specimen also increased after retrofitting. The RC jacket makes the joint and the column more rigid, and the deformations of these components should reduce after retrofitting. For further scrutiny, joint shear deformation is plotted against the applied storey-drift in Figure 4b. As expected, the joint shear deformation is significantly less in the retrofitted specimen than that in the original specimen. It indicates that unlike the original specimen that was about to undergo joint shear failure, the joint may not be responsible for the peak load and eventual failure of the retrofitted specimen. For confirmation, joint shear stress computed assuming perfect bond is plotted against the joint shear strain in Figure 4c. In spite of the high strength concrete used for RC jacketing which certainly enhances the joint shear strength, the maximum joint shear stress induced in the retrofitted specimen (5.5 MPa) is substantially less than that induced in the original specimen (7 MPa). This corroborates that the maximum storey-shear force corresponds to the capacity of the beam, because the column capacity was also improved by RC jacketing. Consequently, the increased deformability of the retrofitted specimen must have come mainly from the plastic flexural deformation of the beam. In spite of the undesirable strong-beam weak-column status of the original specimen, the retrofitted specimen behaved as the favourable strong-column weak-beam type.

**Conclusion**

A full-scale lightly reinforced concrete beam-column sub-assembly was strengthened by casting an RC jacket outside the column and the joint, and the improvement brought over by the retrofitting technique in the cyclic response of the specimen was verified experimentally. The joint of the original specimen was not adequately reinforced to fulfil seismic design requirements, and it was the weakest component of the sub-assembly. When subjected to cyclic lateral loading, the joint panel hence experienced severe damage due to excessive shear deformation while the beam and column remained virtually undamaged. The original specimen was vulnerable to joint shear failure. On the other hand, the retrofitted specimen failed after the formation of a plastic hinge in the beam, and the joint was no longer the weakest component of the sub-assembly. Apart from the increase in the capacity and deformability, the shear deformation of the joint panel reduced significantly after retrofitting. It is concluded that the RC jacketing method is effective in strengthening non-seismic RC frames with inadequately reinforced joints.
Introduction

Under blast loading, the concrete damage and fragmentation result from both impulsive loading by stress waves and gas-driven fracture propagation. Studies tend to indicate that stress waves generated by the detonation of an explosive charge are responsible for the development of damage zone in the concrete material and the subsequent fragment size distribution, while the explosion gases are important in the separation of a crack pattern that has already been formed during the passage of the stress wave. Therefore, modelling the behaviour of concrete materials in the very high strain rate regime plays a crucial role in the modelling of concrete break-up and fragmentation processes.

Concrete is a brittle material. As with most other brittle materials, the strain-rate dependence of its dynamic behaviour can be derived from evolution of damage based on fracture mechanics. This paper presents a model for damage and fracture of concrete materials under dynamic loading. The concrete is assumed to be homogeneous continuum with pre-existing micro cracks. The evaluation of damage is formulated based on the continuum fracture mechanics of micro-crack nucleation, growth and coalescence. The required material constants representing initial crack properties are derived from available material dynamic tests; subsequently, the dynamic stress-strain curves, the fracture strain energy as well as the nominal fragment size can be established.

Model formulation

An empirical function is derived to account for the increase of dynamic Young’s modulus at very high strain rate. Thus, the elastic strain energy can be expressed as

\[ U_0 = \frac{1}{2} e^{\alpha \varepsilon} E(1-D) \varepsilon, \]

while

\[ \sigma = \frac{\partial U_0}{\partial \varepsilon} = e^{\alpha \varepsilon} \frac{E(1-D)}{\varepsilon} \]

where \( E \) is the static Young’s modulus for the undamaged material, \( D \) is a damage scalar varying between 0 to 1, \( \varepsilon \) is the strain rate, \( a \) and \( b \) are constants, and \( e^{\alpha \varepsilon} \geq 1 \).

According to statistical fracture mechanics, the damage scalar \( D \) may be defined as [1]

\[ D(t) = \int_{t_c}^{t} \dot{N}(s)V(t-s)ds \]

where \( t_c \) is the time duration needed for the tensile strain \( \varepsilon \) to reach the critical value \( \varepsilon_c \) [2]. For uniaxial tension

\[ \varepsilon_{cr} = \sigma_{cr}/E. \quad \dot{N} = \alpha(\varepsilon - \varepsilon_{cr})^b, \]

where \( \alpha \) and \( \beta \) are material constants, and

\[ V(t-s) = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi c_s^3(t-s)^3 \]

where \( c_s \) is the crack growth velocity. Substituting the above into Eq. (3), and assuming a constant strain rate,

\[ D(t) = \frac{4}{3} \alpha c_s^3 \varepsilon_c^3 \int_{t_c}^{t} (s-t_c)^b(t-s)^3 ds = m \varepsilon_c^b (t-t_c)^{b+4} \]

where

\[ m = \frac{8\pi c_s^3 \alpha}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)} \]

Hence,

\[ \sigma = e^{\alpha \varepsilon_c} \frac{E\varepsilon_c}{\varepsilon} \quad \text{for} \quad t \leq t_c \]

\[ \sigma = e^{\alpha \varepsilon_c} \frac{E\varepsilon_c}{\varepsilon} \left[ 1 - m \varepsilon_c^b (t-t_c)^{b+4} \right] \quad \text{for} \quad t > t_c \]

Differentiating Eq. (8) with respect to time and setting to zero yields

\[ m \varepsilon_c^b (t_n - t_c)^{b+1} [t_n - t_c + t_n (\beta + 4)] = 1 \]

where \( t_n \) is the time at which maximum stress is attained. The above equation can be used to evaluate \( \beta \) and \( m \) from dynamic strength test data; thus, the dynamic stress-strain relationship can be obtained. The dynamic strain energy upon fracture is then calculated by

\[ U_0 = \int_{t_v} \int_{0}^{\sigma} \alpha d\varepsilon dV \]

By expressing the damage function in terms of the distribution of crack size, the fragment size distribution can be derived; hence the nominal fragment size,

\[ L_n = \frac{6c_s}{\beta + 3} m^{-1/\beta+4} \varepsilon_c^{b/\beta+4} \]

Examples

Based on available test results on concrete and mortar under high strain rates, the coefficients for the dynamic Young’s modulus are evaluated to be \( a = -0.08502 \) and \( b = 0.01441 \). Figure 1(a) shows a set of measured dynamic to static strength ratios for concrete under tension as a function of the log-scale strain rate. Based on these data, the material constants \( \beta \) and \( m \) are determined. Figure 1(b) shows the corresponding dynamic stress-strain curves according to Eqs. (7) & (8). The predicted dynamic strength to static strength ratios are compared with the test data in Figure 1(a).
The predicted fragment sizes for concrete and mortar under different loading conditions are shown in Figure 2(a). Figure 2(b) compares their fracture strain energy.

Conclusion

A model for the dynamic behaviour of concrete materials is proposed based on continuum fracture theory. Based on this model, explicit dynamic stress-strain relationship can be established upon the determination of necessary material constants. Nominal fragment size for concrete under dynamic loading can also be obtained. The calculation of fracture strain energy will be useful in an energy-based concrete fragmentation and debris throw prediction procedure under blast loading.

References


Anisotropic Dynamic Damage of Rock Materials under Blast Loading

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Introduction
In rock blasting, it is generally understood that the stress waves cause significant damage. The propagation, reflection and interaction of the stress waves result in crushing, spalling and fragmentation of rock materials. It is observed that the cracks by blasting are strongly oriented. The influences of the shape, size, orientation and distribution of cracks in a rock mass, which usually result in different material properties in different directions cannot be captured by an isotropic damage scalar. For this reason, an anisotropic damage model is presented for dynamic damage of initially isotropic rock materials under blasting loading.

Constitutive equations
A rock material usually suffers damage by the development of distributed microscopic cracks and leads to the final fracture by their coalescence. To develop a feasible damage theory in this paper, it is postulated that the anisotropic damage state can be described by a damage tensor

\[ D = \sum_{i} D_i n_i \otimes n_i \]  

where \( D_i \) and \( n_i \) are the principal value and the unit vector of principal direction of the tensor \( D \), and \( D_i \) can be interpreted as the ratio of area reduction in the plane perpendicular to \( n_i \) caused by the development of microcracks.

As a point of departure, we postulate that the total strain tensor \( \varepsilon \) can be decomposed into elastic and plastic parts, namely \( \varepsilon = \varepsilon^e + \varepsilon^p \), and the free energy function can be expressed as

\[ \psi(\varepsilon^e + \varepsilon^p, q, C) = \psi_{\text{el}}(\varepsilon^e, C) + \psi_{\text{pl}}(q, C) \]

\[ = \frac{1}{2} \varepsilon^e : C : \varepsilon^e + \psi_{\text{pl}}(q, C) \]

where \( C \) is the damaged elasticity tensor, \( q \) denotes a suitable set of plastic variables. Consequently, the stress-strain relation can be obtained as follows

\[ \sigma = \frac{\partial \psi}{\partial \varepsilon} = C : \varepsilon^e \]  

For the damage-dependent elasticity tensor \( C \), by introducing an intermediate second-order tensor \( \Phi \), it is defined as

\[ C = \lambda \Phi \otimes \Phi + 2\mu \Phi \otimes \Phi \]  

where the symbol \( \otimes \) denotes the dyadic product of two tensors, the symbol \( \otimes \) denotes the symmetrized dyadic product defined as \( (A \otimes B) : C = A : (C + C^\top)/2 : B^\top \). For any arbitrary second order tensors \( A, B, C, \Phi \) is a function of \( D \), and \( \lambda \) and \( \mu \) are Lamé’s constants of virgin material. As the elasticity tensor must satisfy the positive definite condition, the elasticity tensor expressed in Eqn (4) can be reduced to

\[ C = (I - D) \cdot C^+ \cdot (I - D) \]  

under the hypothesis of complementary energy equivalence, where \( I \) is the second-order identity tensor and the isotropic elasticity tensor of the virgin material \( C^+ = \lambda I + 2\mu \Phi \). Substituting Eqns (1) and (5) into Eqn (3) yields

\[ \sigma_i = (I - D) \left[ \alpha (\theta - \xi_i^+) + 2\mu (I - D) \xi_i \right], \]

\( (i = 1, 2, 3, \text{no summation}) \)

where \( \theta = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \), \( \xi_i = D \varepsilon_i + D \varepsilon_2 + D \varepsilon_3 \), and \( \varepsilon_1, \varepsilon_2, \varepsilon_3 \) are principle strains.

Damage evolution
Under blast loading, microcracks will initiate and grow in a rock mass, and damage will be accumulated. As a certain time duration is needed for fracture to take place when a rock material is subjected to a stress higher than its static strength, according to the definition of the damage variable, the evolution of damage in the \( i \)-th principal direction \( (i=1,2,3) \) can be determined by the number of cracks which activate at the time \( t \) as follows

\[ D_i(t) = \frac{1}{2} c_g \int \langle \varepsilon_i - \varepsilon_{i0} \rangle^\beta (t-s)^\alpha ds \]

where \( c_g \) is the time duration needed for fracture to take place, \( \varepsilon_{i0} \) is the quasi-static failure strain, \( \alpha \) and \( \beta \) are material parameters, and \( c_g \) is the crack growth velocity. By studying the dynamic crack propagation, the relation \( c_g = 0.38 \frac{\sqrt{E}}{\rho} \) is usually assumed. The angular bracket \( \langle \cdot \rangle \) denotes a function defined by \( \langle \cdot \rangle = (|\cdot| + x)/2 \).

Identification of the parameters
In the present damage model, four parameters, namely \( \alpha, \beta, c_g \) and \( \varepsilon_{i0} \), need to be determined from the dynamic fracture properties of rock materials. Since most rate-dependent rock tests performed are uniaxial, the static failure strain \( \varepsilon_{i0} \) can be easily determined from uniaxial quasi-static tensile test results. The crack growth velocity \( c_g \) can be obtained by the
above expression. The other two parameters can be determined by using results obtained from the uniaxial tensile test. This is discussed in the following.

In the case of uniaxial tension, a rock material is damaged by the development of distributed microscopic cracks and leads to the final fracture by their coalescence without significant inelastic deformation. Thus for this case, ignoring the plastic strain and assuming the uniaxial strain rate is constant, the fracture stress at a certain strain rate in uniaxial tension can be obtained as

$$\sigma_f = (1 - D_f) \sigma_u + E (1 - D_f) \left( \frac{D_f}{m} \right)^{\frac{1}{\beta + 3}} \dot{\varepsilon}_u^{\frac{1}{\beta + 3}},$$

(8)

where $\dot{\varepsilon}_u$ is the constant strain rate of uniaxial tension, $D_f$ is the damage value when dynamic tensile stress reaches the dynamic fracture stress, and $m = \frac{2\alpha \varepsilon_u}{(\beta + 1)(\beta + 2)(\beta + 3)}$.

Dependence of the fracture stress on strain rate is provided by the above equation. Since fracture stress for many brittle materials such as rock and concrete depends on the cube root of the strain rate, $\beta$ can be taken as equal to 6. The parameter $\alpha$ can be determined by using the predicted dependence of fracture stress on strain rate from Eqn (8) to obtain a fit with the data obtained by experiments. According to the numerical investigations and some test results of rock materials under explosive loading, the damage value is about 0.22 when the dynamic tensile stress reaches the dynamic fracture stress, namely $D_f = 0.22$.

Application

A single borehole blasting experiment was conducted in the nominal 80 ml/kg oil shale. The blasthole was perpendicular to the ground surface with a diameter of 0.162m. The explosive column had a length of 2.5 m and the stemming length was 2.5 m. The detonation point was positioned at the bottom of the explosive column and on the line of symmetry. The representative properties of 80 ml/kg oil shale are: elastic modulus $E = 17.8$GPa, yield strength $\sigma_y = 50$MPa, density $\rho = 2260$kg/m$^3$, Poisson’s ratio $\nu = 0.27$, and quasi-static tensile strength $\sigma_u = 5$MPa.

Using the above oil shale properties and the test data, the material parameters in the model will be determined for oil shale. The parameter $\beta$ is taken to be equal to 6 so that the fracture stress is cube root dependent on the loading rate, and the crack growth velocity $c_g$ is 1066 m/sec that is calculated from the above relation expression. The parameters $\alpha$ is taken to be $\alpha = 1.2 \times 10^{15}$ / m/sec that is determined by fitting Eqns (8) to the test data shown in Figure 1.

The present damage model is implemented into the commercial program AUTODYN2D as its user subroutine. The Mohr-Coulomb criterion is adopted. The region of significant rock damage in the oil shale is usually measured by excavating the rock loosened by the blast. The crater formed in the experiment was excavated and its dimensions surveyed. The radius of the crater is about 4.9m, the depth of the crater on the line of symmetry is about 1.75m. This is compared to the calculated rock damage regions defined by the contour in the material where the damage scalar $\Omega$ exceeds 0.22. The damage scalar $\Omega$ is defined as $\Omega = \min D_i(t)$, and here we assume that the rock is loosened enough to be excavated when the damage scalar $\Omega$ exceeds 0.22.

Conclusion

By employing a second rank symmetric damage tensor in the irreversible thermodynamic frameworks, a model for dynamic anisotropic damage of rock materials under explosive loading has been developed. It considers the experimental facts that a brittle material does not fail if the applied stress is lower than its static strength and certain time duration is needed for fracture to take place when it is subjected to a stress higher than its static strength.

Based on the mechanics of microcrack nucleation, growth and coalescence, the evolution of damage is formulated. The various damage activation parameters involved in the model can be easily determined. The model has been calibrated by a field blasting test.
Numerical Simulation of Oblique Perforation of Concrete Slab by Hard Projectile

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Introduction

‘Hydrocodes’, which are actually computer coding for solving wave-propagation problems, have become more sophisticated, and its use in the investigation of stress-wave propagation in fluid and solids is increasing. In the commonly used hydrocodes, numerical models are available to simulate many physical phenomena including the highly dynamic events – perforation of concrete panel by hard projectile. However, constitutive modelling for concrete material remains the major hindrance when it comes to obtaining accurate predictions. This article presents our endeavour in developing a dynamic plastic-damage material model for concrete, and its application in simulating the perforation of a steel projectile through concrete panels at 30 degrees obliquity. The simulation results are in reasonably good agreement with the experimental data.

Constitutive model for concrete material

A complete constitutive model comprises pointers and rules. The pointers demarcate the transition between two distinct behaviours, and the rules govern the characteristics of the behaviour. The elastic limit, the ultimate strength limit and the residual strength have been identified as the demarcating pointers, which mark the transitions from elastic to plastic and then to partially damaged behaviour. Among those pointers and governing rules, many empirical formulae or postulates can be found in literature. A distinct feature in our current study lies in its semi-empirical/semi-theoretical way of deriving the pointer for the ultimate strength in the 3-dimensional stress space (namely π-space). The ultimate strength limit is established firstly by adopting two meridian r-ξ curves (tensile at θ=0° and compressive θ=60°) as skeleton, and then being completed via the twin-shear strength theory. Empirical meridians such as Kotsovos’ can be expressed in Westergaard coordinates (ξ-r-θ), i.e.

\[
 r_e = \frac{r_r \sin(60°)}{\sin(\theta) + \frac{r_r \sin(60° - \theta)}{\cos(\theta)}} (1- b) + \frac{r_r}{\cos(\theta)} \quad \text{when} \quad 0° \leq \theta \leq \theta^*_b
\]

\[
 r_s = \frac{r_r \sin(60°)}{\sin(\theta) + \frac{r_r \sin(60° + \theta)}{\cos(\theta)}} (1- b) + \frac{r_r}{\cos(\theta)} \quad \text{when} \quad \theta^*_b \leq \theta \leq 60°
\]

where \( \theta^*_b = \arctan \left( \frac{1}{\sqrt{3}} \left( \frac{2 r_s}{r_r} - 1 \right) \right) \).

The governing rules for the non-elastic behaviour define the stress path between the elastic limit and the ultimate strength limit. The strain-hardening effect is characterized by an effective plastic strain \( \varepsilon_p^\prime \), which is used as a parameter to define the stress-strain relationship in that plastic regime.

The governing rule for the post-ultimate strength stage defines the erosion of strength. It is characterized by a damage scalar \( D \), which accounts for the accumulated equivalent plastic strain \( \Delta \varepsilon_p \), i.e.

\[
 D = \sum \frac{\Delta \varepsilon_p}{\varepsilon_p^\prime}
\]

where \( \varepsilon_p^\prime \) denotes the total plastic strain to fracture under a constant pressure and \( \varepsilon_p^\prime \) is a function of two parameters \( (D_1 \text{ and } D_2) \). Value of \( D \) varies between 0.0 and 1.0. When \( D = 1 \), it represents the fractured state having the minimum residual strength.

Numerical Simulation

The above-mentioned constitutive model for concrete is employed to simulate the oblique perforation process of a concrete slab by a hard projectile. The impact configuration is similar to those in the experimental tests carried out by Buzaud at al.(1999). The concrete slab is 3m square and has a thickness of 600mm. The high-strength steel projectile impacts at the centre of the concrete slab at a velocity of 338m/s with a 30° angle of incidence.

Figures 1 and 2 show, respectively, the geometric configuration of the projectile and the reinforced concrete target. The projectile was machined out of a 35NCD16 high strength steel rod with an elasticity limit of 1300MPa. Having a total length of 960mm and a diameter of 160mm, its nose is tangent ogive shape with a Calibre Radius Head (CRH) of 6. The target was reinforced by
two flexural reinforcement meshes at a distance of 50mm from the front and rear faces, with $\phi 16 \text{ mm}$ high-adherence steel bars. The steel-to-concrete area ratio is 0.67%, which falls in the category of light-reinforcement. Against this background, the reinforcement bars are not included in the present simulation. Table 1 lists the material constants for the concrete material. Equation of state used here is a piece-wise linear porous model. Four density pressure pairs are set here. Erosion limit for incremental geometric strain is set at 150%.

The material statuses after perforation are shown in Figures 3 and 4. Figures 5 and 6 depict the experimental results for the damage area in the front and rear face respectively. Comparing the numerical results with the experimental data shows that: i) regarding the damaged areas in the front (Fig.3) and the rear faces (Fig.4), numerical results agree well with the experimental ones; ii) regarding the damage initiation point at the rear face, numerical prediction also agrees well with the experimental data; iii) regarding the exit point on the rear face, numerical simulation (Fig.4) also indicates similar location as the experiment (Fig.6).

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Regarding the exit velocity, numerical result is 214m/s, which is slightly higher than the experimental result of 180m/s. It could be partly due to the omission of steel reinforcement bars in the numerical simulation.

### Table 1. Typical values of material parameters for the concrete model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Density</td>
<td>2454.2kg/m³</td>
</tr>
<tr>
<td>Porous Sound Speed</td>
<td>2693.17m/s</td>
</tr>
<tr>
<td>Solid Sound Speed</td>
<td>2693.17m/s</td>
</tr>
<tr>
<td>Compressive strength</td>
<td>48MPa</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>4.0MPa</td>
</tr>
<tr>
<td>Density #1</td>
<td>2368.0kg/m³</td>
</tr>
<tr>
<td>Density #2</td>
<td>2378.0kg/m³</td>
</tr>
<tr>
<td>Density #3</td>
<td>2411.0kg/m³</td>
</tr>
<tr>
<td>Density #4</td>
<td>2446.5g/m³</td>
</tr>
<tr>
<td>Pressure #1</td>
<td>0MPa</td>
</tr>
<tr>
<td>Pressure #2</td>
<td>44MPa</td>
</tr>
<tr>
<td>Pressure #3</td>
<td>180MPa</td>
</tr>
<tr>
<td>Pressure #4</td>
<td>333MPa</td>
</tr>
<tr>
<td>Shear Modulus</td>
<td>8.40GPa</td>
</tr>
<tr>
<td>$b$</td>
<td>0.6</td>
</tr>
<tr>
<td>$E$</td>
<td>20GPa</td>
</tr>
<tr>
<td>$D_1$</td>
<td>0.03</td>
</tr>
<tr>
<td>$D_2$</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### Conclusion

The present dynamic plastic-damage model can yield reasonably accurate predictions for the oblique perforation process, in particular for concrete panel impacted by a steel projectile. The distinct feature is its novel way of constructing the ultimate strength limits. The coding is tied to the available hydrocode AUTODYN3D (2000). Not only the velocity history of projectile, but also the damage zones and the overall performance can be obtained with an acceptable degree of accuracy.
On Modelling of Explosive Wave Input and Propagation through Jointed Rock

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Introduction

Very often, explosive waves are considered as shock waves and its transmission to the rock mass (Fig.1) are in the form of pressure waves. Modelling of the pressure wave input has two different approaches. The most common one is to impose the pressure-history at the incident boundary. Another one is to impose the velocity-history on the incident boundary. Our current investigation found that their consequences on jointed rock were not the same. An illustrative example is shown below. A pressure wave is transmitted from one end of a one-dimension rock bar and then it propagates through a joint and reaches the other end. Numerical results for this simple model are obtained using the available computer code UDEC (Universal Distinct Element Code). The two approaches yield markedly different results, in particular the velocity histories at the far end of the rock bar.

Effects on the incident boundary

In fact, the two approaches could yield the same result if the pressure wave travels through a perfect elastic medium without any joints inside. It is because an equivalent stress history at the incident boundary can be derived from the velocity history. Waves travelling in the perfect elastic medium experience no distortion. However, the scenario is totally different in jointed rock mass, even if the rock mass remains perfectly elastic. The existence of a joint or joints in the rock changes the resulting wave pattern, which is the superposition of the incident wave with reflective wave. It wipes off the simple equivalence between the stress history and velocity history at the incident boundary. Actually, imposing the velocity history on the incident boundary is a kind of displacement-constraint, in the sense of dynamic specific displacement. When the velocity of reflective wave is not in phase with the incident one, the effective stress at the incident boundary will no longer be a simple function of the incident velocity wave. In other words, the resulting stress history there will be different from the hypothetical incident pressure waves. It leads to different results at locations downstream.

Example

To fend off possible disturbance arising from other factors, we consider a simple P-wave propagating through a one-dimensional rock bar. Two scenarios are considered. One is a perfectly elastic rock bar without any joints inside. Another is the same rock bar with a transverse joint at the mid-length. The consequences of two approaches in these two scenarios are investigated. The geometrical and material parameters of the rock mass are: length=10m, density $\rho=2650\text{kg/m}^3$, bulk modulus $K=44.1\text{GPa}$, shear modulus $G=33.8\text{GPa}$. Properties of the transverse joint are: normal stiffness $k_n=100\text{GPa/m}$, shear stiffness $k_s=100\text{GPa/m}$, cohesion $C=2\text{MPa}$, friction angle $\phi=35^\circ$, tensile strength $\sigma_t=0\text{MPa}$. At the far end, viscous boundaries without damping are assumed. In the first approach, a train of sinusoidal velocity P-waves is transmitted to the left incident end. The amplitude of velocity wave is 0.065m/s having a frequency of 200Hz, and the whole applied duration is 0.02s. In the second approach, a train of sinusoidal pressure P-waves are transmitted to the incident end. The equivalent pressure wave is derived from the same velocity wave of the first approach as follows.

$$\sigma = \rho C_p v_e$$  \hspace{1cm} (1)

in which $C_p = \sqrt{(K+4G)/\rho}$ denotes the speed of P-wave. Substituting the values of $K$, $G$ and $\rho$ into Eq. 1 yields the amplitude (=1MPa) of the pressure wave. The computational model is shown schematically in Figure 2. Along the length of the rock bar, the dynamic responses at 10 locations (5 before and 5 after the joint if present) are scrutinized.
In the first scenario, no joint is present. The results are as expected. Almost no distortions in the waveform are observed in both approaches. The velocity waveform reached the far end $B_5$ remains the same as the incident waveform.

In the second scenario, a transverse joint is located at the mid-length of the rock bar. The results of the two approaches show markedly differences. In the case of velocity-history input, the tensile parts (negative) of the waveform diminish to negligible undulations immediately after crossing the transverse joint, whilst part of compressive waveform (positive) survives. At the far end $B_5$, it emerges with only two positive half-waves. It is depicted in Figure 3, which shows the velocity histories at location $A_1$, near the incident boundary and also at location $B_5$ at the far end. We note that the incident waveform becomes distorted at $A_1$ soon after entering the incident boundary and is transformed into a total tensile wave (negative) at late time. The mean velocity drifts away from the initial zero value. It is due to superimposition of the incident and reflected waves.

Furthermore, it is worth noting that the two approaches yield very different maximum axial displacements along the rock bar as shown in Figure 5. At locations to the left of the joint, the maximum axial displacement is negative (away from the joint), and the magnitude is much higher in the case of pressure-velocity input.

**Conclusion**

Two seemingly equivalent approaches of transmitting explosive pressure wave into jointed rock mass are investigated. Very often, it is disguised by the scenario, when it occurs in a perfectly elastic medium, which has no discontinuities inside. However, the two approaches yield very different results in jointed rock. The numerical example reveals that the approach using velocity-history input is able to model the incident boundary behaviour at all times. On the other hand, the approach using pressure-history input leads to distorted incident boundary behaviour.
In recent years, the flat plate floor system has been increasingly adopted for high-rise residential buildings in Singapore. However, the available design procedures that have been included in various codes of practice do not cover the more modern types of flat plate floors in which the column layouts or spacing may be irregular and the column cross sections can be very elongated (rectangular). A large opening may also be present next to a column.

For this reason, a joint research has been conducted by the Building and Construction Authority (BCA) and Nanyang Technological University (NTU) to find a practical solution for the structural design of these types of flat plate floors. This paper summarises one part of the joint research programme and focuses on the resulting recommendation for punching shear design of slabs with openings and supported on rectangular columns. The recommended design method is an extension to the BS8110 procedure.

Existing BS8110 method

Compared with other codes, the existing BS8110 method seems to be able to predict the punching failure load of slabs with square column fairly accurately. For reinforced concrete slabs, the available nominal shear strength can be calculated from

\[ v_c = 0.79 \left( \frac{100A_s}{bd} \right)^{1/3} \left( \frac{400}{d} \right)^{1/4} \left( \frac{f_{cs}}{25} \right)^{1/3} / \gamma_m \ (\text{N/mm}^2) \]  

(1)

where \( A_s / bd \) is the percentage of flexural reinforcement, \( 400 / d \) is the size effect factor which should not be taken less than 1.0, \( f_{cs} \leq 40 \text{ N/mm}^2 \) \( \gamma_m = 1.25 \). The ultimate punching shear, \( v \), due to a concentrated ultimate load can then be obtained from

\[ v = V_{ud} \]  

(2)

where \( V_u \) is the ultimate design shear force, \( u \) is the critical (control) shear perimeter, and \( d \) is the effective depth of slab. The BS8110 uses a shear perimeter at 1.5\( d \) away from the face of the column. For a rectangular column, the centre portion on the longer side of the shear perimeter may not be effective in transferring shear stresses, and therefore, the shear perimeter \( u \) should be computed according to Figure 1.

If the calculated shear stress, \( v \), does not exceed, \( v_c \), then no shear reinforcement is needed. The nominal design shear stress \( \tau \) is calculated from Eq. (2), that is \( \tau = V_{ud} / \gamma_m \). For applications to flat plate structures, the design shear force should be calculated using the design effective shear force, \( V_{eff} \), in accordance with Clause 3.7.6 of the BS8110 Code.

Thus, \( V_u = V_{eff} \)

Recommended design procedure: an extension to BS8110

This design procedure is an extension to the BS8110 to make it applicable to punching shear design involving rectangular columns with an opening near the column. To accommodate column rectangularity, a reduction in the control shear perimeter is introduced in the similar way as the Eurocode-2 procedure, except that a longer effective shear perimeter will be used, compared to that recommended by the Eurocode-2. When an opening is present, the radial effective shear perimeter will not necessarily originate from the centre of the column but from the centre of the corner portion of the column. When the column cross section is square, the procedure reduces exactly to the BS8110 method.

Note that the recommended procedure is intended for slab/interior column connections where unbalanced moment is negligible. The complete procedure is described below.

Design procedure for slabs without shear reinforcement

The shear capacity should be checked on a shear perimeter 1.5\( d \) from the face of the column. For a rectangular column, the centre portion on the longer side of the shear perimeter may not be effective in transferring shear stresses, and therefore, the shear perimeter \( u \) should be computed according to Figure 1.

If the calculated shear stress, \( v \), does not exceed, \( v_c \), then no shear reinforcement is needed. The nominal design shear stress \( v \) is calculated from Eq. (2), that is \( v = V_{ud} / \gamma_m \). For applications to flat plate structures, the design shear force should be calculated using the design effective shear force, \( V_{eff} \), in accordance with Clause 3.7.6 of the BS8110 Code.

Thus, \( V_u = V_{eff} \)

Figure 1. Recommended effective shear perimeter
Modification of effective shear perimeter to allow for holes

When an opening in the slab is located at a distance less than six times the effective depth of the slab from the edge of a column or a concentrated load, then the shear perimeter $u$ should be reduced. That part of the shear perimeter which is enclosed by the radial projections from the centroid of the corner portion of the column to the edge of the openings should be considered ineffective (see Figure 2). The projection lines from one end of a column should not cross the centre line of the column.

Design example

The flat-plate floor of Figure 3 is supported by rectangular columns of 200 mm by 1000 mm in cross section and an opening of 200 mm by 400 mm has been introduced next to the slab-column connection C2 as shown in the figure. The imposed live load is 3 kN/m², concrete strength $f_{cu}$ is 40 N/mm², steel yield strength $f_y$ is 460 N/mm² (Check the safety of the connection C2 against punching).

Solution

The design ultimate load is $w_u$

$w_u = 1.4 g_c + 1.6 q_l = 1.4 (0.15 * 24) + 1.6 (3) = 9.84 \text{ kN/m}^2$

The total design load $F$ on a typical panel adjacent to the connection C2 is

$F = 9.84 * 4 * 5 = 196.8 \text{ kN}$

At the column face the design shear force is $V_u = V_{eff} = 1.15 V_l$

$V_u = V_{eff} = 1.15 * 196.8 = 226 \text{ kN}$

The shear stress $v_{max}$ at the column face is then equal to

$v_{max} = V_u / u_d = 226 * 10^3 / \{2 * (200 + 1000) * 112\}$

$= 0.84 \text{ N/mm}^2 < 5 \text{ N/mm}^2$ or $0.8 (f_{cu})^{1/2}$ O.K.

The top flexural reinforcement in the column strip $A_{top}$ has been calculated to be $8T13$ or 1062 mm² in each direction.

The critical shear perimeter is at $1.5d$ from the column face. For connection C2, the effective shear perimeter $u$ should be determined according to the recommended procedure shown in Figure 2. For convenience, the detail of connection C2 is shown in Figure Ex-2. The effective shear perimeter $u$ is given by the sum of the effective parameters on each side of the column, that is, $u = u_e + u_w + u_n + u_s$, where the subscripts e, w, n, s indicate respectively the effective values on the east, west, north, and south of the column.

Note that $c$ should be taken as the smaller of three values, that is,$c \leq \begin{cases} \frac{a}{2} & = 1000/2 = 500 \text{ mm} \\ 2b & = 2 * 200 = 400 \text{ mm} \\ 5.6d - b/2 & = 5.6 * 112 - 200/2 = 527 \text{ mm} \end{cases}$

Thus, $c = 400 \text{ mm}$, and

$u_e = 1.5d + b/2 + y$

$= 1.5d + b/2 + (100/300) * (1.5d + 100) = 1.5$

$(112) + 200/2 + 89.3$

$= 357 \text{ mm}$

$u_w = 1.5d + b + 1.5d = 1.5*112 + 200 + 1.5*112$

$= 536 \text{ mm}$

$u_n = 1.5d + c + c - x$

$= 1.5d + 400 + 400 - (100 / 500) * (400 - 1.5d)$

$= 168 + 400 + 400 - 46.4$

$= 922 \text{ mm}$

$u_s = 1.5d + c + c + 1.5d = 168 + 400 + 400 + 168$

$= 1136$
The total effective shear perimeter $u$ is then given below
\[ u = u_e + u_w + u_n + u_s = 357 + 536 + 922 + 1136 = 2951 \text{ mm} \]

The design shear force $V_u$ is equal to $V_{eff}$, that is
\[ V_u = 1.15 \times \{5 \times 4 - (0.536) \times (1.336)\} = 218.2 \text{ kN} \]

The shear stress $v_u$ is then equal to
\[ v_u = 218.2 \times 10^3 / (2951 \times 112) = 0.66 \text{ N/mm}^2 \]

The concrete shear strength $v_c$ can be calculated according to the following formula:
\[ v_c = 0.39 \times \left( \frac{100A}{bd} \right)^{1/3} \left( \frac{400}{d} \right)^{1/4} \left( \frac{f_{ck}}{25} \right)^{1/3} / \gamma_n \quad (\text{N/mm}^2) \text{ or} \]
\[ v_c = 0.79(0.747)^{1/3} \left( \frac{400}{112} \right)^{1/4} \left( \frac{400}{25} \right)^{1/3} / 1.25 \]
\[ = 0.792 \text{ N/mm}^2 > 0.66 \text{ N/mm}^2 \Rightarrow \text{O.K.} \]

Since $v_u$ is less than $v_c$, the connection is safe and shear reinforcement is not needed.

**Closure**

A recommendation on the punching shear design of concrete slabs with openings and supported on rectangular columns has been presented. The recommended procedure has previously been shown to be reliable and accurate. A total of 134 slab specimens including 20 new slab specimens tested under BCA-NTU joint research programme have been used to verify the accuracy and consistency of the recommended procedure. Engineers can now use the recommended procedure in engineering practice with confidence.

**Reference**

[1] Punching shear strength on slabs with openings and supported on rectangular columns, Final Report, BCA-NTU joint research on flat plate structures – Phase 1A, School of Civil and Environmental Engineering, Nanyang Technological University, 2001.
Widths of design strips

The transverse width of the design strip, \( l_2 \) can be defined as follows (Figure 2):
- The midspan (average) width of the strip bounded laterally by the centrelines of adjacent panels on each side of the column line.
- For an edge or perimeter strip, \( l_2 \) should be taken as the mid-span width between the edge of panel to panel centrel ine.

The width of the column strip can also be taken approximately as half the width of the design strip.

Stiffness of transverse torsional member

For irregular slab geometry, the ACI [1] formula for \( K_t \) will have to be modified to take into account the irregular geometry, especially at edge slab-column connections (Figure 3). Therefore, a reduction factor \( \alpha_t \) is introduced, depending on the slab geometry, to modify the torsional stiffness of the transverse torsional members. The coefficient \( \alpha_t \) is listed in Table 1. The modified torsional stiffness of the transverse torsional members \( K'_t \) can then be computed from the following expression:

\[
K'_t = \sum \alpha_t K_t \tag{3}
\]

where:

\[
K_t = \frac{9E_t C}{l_t^4 (1 - c^2_1/l^2)}; \quad C = \left( 1 - 0.63 \frac{x}{y} \right) \frac{x^3 y}{3/2} \quad \text{(torsional stiffness)}
\]

Distribution of negative and positive moments

The basic ACI values of the negative and positive moments for an interior span are 0.65\( M_0 \) and 0.35\( M_0 \), respectively, where \( M_0 \) is the total static moment.

For an end span, the distribution of moments is thus expressed as:

\[
M_{in} = M_0 \left[ 0.75 - \frac{0.10}{1 + (1/\gamma^* \beta)} \right] \tag{4}
\]

\[
M_{m} = M_0 \left[ \frac{0.63 - 0.28}{1 + (1/\gamma^* \beta)} \right] \tag{5}
\]

\[
M_{ex} = M_0 \left[ \frac{0.65}{1 + (1/\gamma^* \beta)} \right] \tag{6}
\]

where \( M_{in} \) is negative interior support moment, \( M_m \) is midspan positive moment, \( M_{ex} \) is negative exterior support moment, and \( \alpha_c = K_{ec}/K_s \).

For a span bounded by two exterior supports, the distribution of the three moments are:

\[
M_{ex1} = M_0 \left[ \frac{0.65}{1 + (1/\gamma^* \beta)} \right] \tag{8}
\]

\[
M_{in} = M_0 - (M_{ex1} + M_{ex2})/2 \tag{9}
\]

\[
M_{ex2} = M_0 \left[ \frac{0.65}{1 + (1/\gamma^* \beta)} \right] \tag{10}
\]

Moments in perimeter strips with an irregular end support

In an irregular flat plate floor, the end support condition can be irregular, that is, the angle \( \theta \) can vary from 0° to 90° (Figure 4).

<table>
<thead>
<tr>
<th>( \gamma^* )</th>
<th>( \alpha_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0.00</td>
</tr>
<tr>
<td>30°</td>
<td>0.25</td>
</tr>
<tr>
<td>60°</td>
<td>0.75</td>
</tr>
<tr>
<td>≥ 90°</td>
<td>1.00</td>
</tr>
</tbody>
</table>

* Angle between slab edge and span direction (Fig. 3)
The distribution of the three moments over support 1, mid-span, and support 2 can be expressed by the following equations

\[ M_{\text{spl}} = M_{\text{cr}} + \alpha_r (M_{\text{m1}} - M_{\text{m2}}) \]  
\[ M_{\text{m2}} = \frac{(M_{\text{spl}} + M_{\text{spl}})}{2} \]  
\[ M_{\text{m2}} = M_{\text{cr}} + \alpha_r (M_{\text{m2}} - M_{\text{cr}}) \]  

The values of \( \alpha_r \) for various values of \( \theta \) are listed in Table 2.

<table>
<thead>
<tr>
<th>( \theta^\circ )</th>
<th>( \alpha_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \leq 0^\circ )</td>
<td>0.00</td>
</tr>
<tr>
<td>30(^\circ)</td>
<td>0.25</td>
</tr>
<tr>
<td>60(^\circ)</td>
<td>0.75</td>
</tr>
<tr>
<td>( \geq 90^\circ )</td>
<td>1.00</td>
</tr>
</tbody>
</table>

* see Figure 4 for definition of \( \theta \).

Distribution of moments to column and middle strips

Naturally, for an irregular support condition (\( \theta \) in between \( 0^\circ \) and \( 90^\circ \)), the percentage of negative moment to be distributed into the column strip should lie between 75\% and 100\%. The values in Table 3 are suggested and used here. The distribution of the positive span moment into the column and middle strips follows the ACI values of 60\% and 40\%, respectively.

<table>
<thead>
<tr>
<th>( \theta^\circ )</th>
<th>% ( M ) for column strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \leq 0^\circ )</td>
<td>100%</td>
</tr>
<tr>
<td>30(^\circ)</td>
<td>95%</td>
</tr>
<tr>
<td>60(^\circ)</td>
<td>80%</td>
</tr>
<tr>
<td>( \geq 90^\circ )</td>
<td>75%</td>
</tr>
</tbody>
</table>

* see Figure 4 for definition of \( \theta \).

Effective moment of inertia

The effective moment of inertia \( I_e \) formula [1, 2] for a flexural member is given below by:

\[ I_e = \left( \frac{M_{\text{cr}}}{M_a} \right)^{\frac{3}{2}} I_g + \left[ 1 - \left( \frac{M_{\text{cr}}}{M_a} \right)^{\frac{3}{2}} \right] I_{cr} \]  

(14)

where \( M_{\text{cr}} \) is the cracking moment, \( M_a \) is maximum applied moment, \( I_{cr} \) is the fully cracked moment of inertia, \( f_c \) is the modulus of rupture, and \( y \) is the centroidal distance to the outermost tension fibre.

For flexural members with monolithic or continuous supports such as flat plate floors, an average effective stiffness [2] can be calculated as follows:

\[ E_{\text{c}} I_{c, \text{avg}} = E_{\text{cr}} I^\circ \left[ 1 - \left( \frac{M_{\text{cr}}}{2 M_a} \right)^{\frac{3}{2}} \right] + E_{\text{c1}} I_{c1} + E_{\text{c2}} I_{c2} \left( \frac{M_{\text{cr}}}{2 M_a} \right)^{\frac{3}{2}} \]  

(15)

where \( E_{\text{c}} \), \( I_{c, \text{avg}} \) and \( I_{c1} \) is the effective moment of inertia at midspan, ends 1 and 2, respectively; \( M_{\text{cr}} \) and \( M_{a} \) are, respectively, the negative moments at ends 1 and 2.

The magnitude of deflection in the middle of the span can be obtained by:

\[ \Delta_m = \frac{5 L^2}{48 E_c I_{c, \text{avg}} \left( M_a + \frac{1}{10} (M_{c1} + M_{c2}) \right)} \]  

(16)

Effective Modulus of Rupture of Concrete

Scanlon and Murray [3] suggested a useful formula for \( f_c \) as shown in Eq. (17). The value of \( f_c \) obtained from Eq. (17) will be used in the calculation of the cracking moment in this paper.

\[ f_c = 0.33 \sqrt{f_e} \]  

(17)

Reduced Modulus of Elasticity of Concrete

When the deflection beyond the serviceability limit state is required, more accurate results can be obtained by reducing the \( E_c \) into \( E_r \) as follows:

\[ E_r = \frac{\left( \frac{2 M_{c1}}{M_a} \right)^{\frac{3}{2}} E_c + \left[ 1 - \left( \frac{2 M_{c1}}{M_a} \right)^{\frac{3}{2}} \right] E_{\text{emin}} \}}{E_c} \]  

(18)

where: \( E_c = \) secant modulus of elasticity of concrete; \( E_{\text{emin}} = 0.3 E_c \); \( M_{c1} = \) cracking moment; \( M_a = \) moment in the slab at the section considered.

Procedure for Computing Deflections of Irregular Flat-Plate Floors

The step-by-step procedure is as follows:

1. Define the transverse width of the design strip, \( l_2 \) (Figure 2).
2. Determine the clear span, \( l' \), and the modified column dimensions, \( c'_{12} \) (Figure 1).
3. Define the width of column strip (Figure 2).
4. Compute the total static moment \( M_0 \) (using \( l' \) as clear span).
5. Calculate the reduced torsional stiffness of the torsional members, \( K' \) (Eq. 3).
6. Determine the negative and positive moments in a span (use Eqs. 4 to 6 or 8 to 10 for internal strips; and Eqs. 11 to 13 for perimeter or edge strips).
7. Distribute the negative and positive moments to the column and middle strips (use Table 3).
8. Calculate the cracking moments of the slab sections (use \( f_c \) of Eq. 17 for modulus of rupture).
9. Calculate the fully cracked transformed moment of inertia of the slab $I_c$ and the effective moment of inertia $I_e$ (Eq. 14).

10. Compute the average effective moment of inertia, $E_I_{e,avg}$ (Eq. 15). Use $E$ of Eq. 18, especially for higher load level.

11. Calculate the midspan deflection $\Delta_m$ (Eq. 16).

**Experimental verification**

To verify the reliability of the proposed method, two sets of experimental results involving two large-scale flat plate floor tests will be used. One of the floor slabs is a 7-column irregular flat plate floor of 6.3 m long, 5.1 m wide, and 0.1 m thick [4]. The other specimen is a 14-column irregular flat plate floor slabs of 11.75 m long, 6.7 m wide, and 0.1 m thick [5]. The floor specimens were tested to failure at NTU, Singapore.

Figure 6 compares the computed against measured deflections at load levels of 80% of the ultimate load for the 7-column and 14-column specimens. It can be seen that most of the computed values lie within the 15% range of the measured values.

**Conclusion**

From the study and discussion above, the following conclusions can be made:

1. The effective moment of inertia concept, together with the Direct Design Method, can be used for computing deflections of irregular flat plate floors.

2. At higher load levels, beyond the serviceability limit state, the use of $E_r$ will improve the predicted deflections considerably.

**Acknowledgments**

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**References**


Nonlinear Cracked Analysis of Concrete Deep Beams by Lattice Model

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Introduction

Researchers have proposed a number of simulation models (Teng and Wang, 2001) for concrete behaviour, ranging from simplified models to more complicated models involving multi-dimensional analysis of structural concrete. Some do not model the material continuously but replace the continuum by an array of discrete elements in the form of particles, trusses, or frames in such a manner that the displacement are defined only at the centre of particles, or at the nodes of the trusses or frames. One of the methods that reduce the basic element of concrete to trusses, in a simplified manner that involves computational process and mathematical analysis, is called the lattice model. Although truss element is employed in a lattice model, it would be able to give good predictions since the accuracy of its analysis will largely depend on the characteristics of the adopted constitutive laws together with their failure criteria. The aim of this research is to investigate the feasibility of using a lattice model to perform a nonlinear analysis of concrete deep beams.

Methodology

Size Effect and Effective Width of Struts

In this lattice model, the authors propose that concrete aggregates and steel reinforcement be modelled as a uniaxial truss element. Naturally, convergence of the overall structural behaviour has to be achieved using a reasonable number of truss elements. As the number of truss elements used in the model increases, the length of each truss element decreases. As the truss elements become shorter, the width of each element becomes an important factor in determining the ultimate strength of the concrete members. From the convergence analysis (Figure 1), it was clear that the strength of deep beam is influenced by the size effect factor, \( \lambda \). A widely used value of \( \lambda \) for reinforced concrete members is given by

\[
\lambda = \left( 1 + \frac{d}{d_0} \right)^{0.25}.
\]

In using the lattice method, it becomes necessary to include the size effect factor automatically. Thus, the average effective width of the truss element throughout its length, which can normally be taken as one-third of its member length (see MacGregor, 1997), should take into account the size effect factor to become width/\( \lambda \).

Constitutive laws

Concrete Compression Member

Once cracking occurs under a biaxial tension-compression loading, the strength and stiffness of the concrete under compression parallel to the crack is reduced due to the lateral softening effect. The proposed lattice model only considers the strain along the concrete strut itself without taking into account the transverse strain caused by cracks between struts. With this approach, the principle transverse tensile stress will be quite low, which does not represent the true behaviour of concrete. To correct this possible problem, another way of accounting for the effect of softening is needed, such as one recommended by ACI 318, which uses efficiency factors instead of softening coefficient. By using the efficiency values given in the code for struts cracked longitudinally due to bottle shaped stress fields without transverse reinforcement, the following effective compression strength is applied to Hognestad’s model for concrete compression members:

\[
\sigma_{2} = \sigma_{\text{eff}} \left\{ \frac{2}{E_c} \left( \frac{\epsilon_2}{\epsilon_c} \right)^2 \right\} ;
\]

\[
\sigma_{\text{eff}} = 0.65 \left( 0.55 + \frac{15}{(f'c)^{1.2}} \right) f'.
\]

Concrete Tension Member

In this study, tensile stresses of concrete play a major role in predicting the load response of a member subjected to shear. Tensile stresses in the diagonally cracked concrete contribute to the shear resistance of the cracked concrete. The softened truss model constitutive law was proposed in this micro lattice modeling. It is described by one linear equation for the pre-cracking stage and one non linear equation for the post-cracking stage (Mau and Hsu, 1989) as follows:

\[
\sigma_1 = E_c \epsilon_1 ; \quad \epsilon_1 \leq \epsilon_u
\]

\[
\sigma_1 = \sigma_{\text{1cr}} \left( \frac{\epsilon_1}{\epsilon_u} \right)^{0.4} ; \quad \epsilon_1 > \epsilon_u
\]

where \( E_c = -2f'c / \epsilon_u \) with \( f'c \) is taken as \(-0.0022\)

\[
\sigma_{\text{1cr}} = 7.5 \left( f'c \right)^{1/2} \text{ (psi)}
\]

as recommended by ACI 318

Steel Member

Since steel reinforcement elements in concrete construction are mostly one-dimensional, it is not necessary to introduce the complexities of multiaxial constitutive relationship for steel. For simplicity, a simple bilinear model is used, as follows:

\[
\sigma_s = E_s \epsilon_s ; \quad \epsilon_s \leq \epsilon_y
\]

\[
\sigma_s = f_s ; \quad \epsilon_s > \epsilon_y
\]

Computer programme

A micromechanics-based lattice model for plain concrete has been proposed and translated into computer programme by Li and Teng (2002). The programme has been further modified here for reinforced concrete deep beams and is outlined in Figure 2.

Results and observations

Two past experimental deep beams (Teng et al., 1996) were modelled using the proposed lattice model, adopting the
Initially, a fine flexural crack was formed in each of the deep beams in the region of maximum bending moment, which is at the mid-span of the beam (Figure 3a and 4a). The load corresponding to the first flexural crack is termed flexural cracking load, $P_f$. This load corresponds to approximately 25% of the ultimate failure load, $P_u$. As the load was increased, the flexural cracks propagated upwards and more flexural cracks were formed. The initiation of the flexural cracks at the beginning was also confirmed in the experiment.

At a load of $P_d$, approximately 50% of $P_u$, the first major diagonal crack was formed (Figure 3b and 4b). This diagonal crack started to form at about one-third of the beam depth from the beam soffit and then propagated very quickly throughout the depth of the deep beam. Similar behaviour was also shown in the experiment.

As the load was further increased, these diagonal cracks extended outwards, both towards the load and towards the support (Figure 3c and 4c). This trend continued until the
ultimate load of deep beams was achieved, accompanied by a significant drop in the load-carrying capacity of the beam (Figure 3g and 4g). The same behaviour also occurred in the experiment, which confirmed the prediction of the model.

Modes of Failure
Mode of failure was identified when the load carrying capacity of the model was about to reach zero value. It can be seen from Figure 5 that both the beam specimen failed due to the formation and propagation of a critical diagonal crack. It was noted that this critical diagonal crack occurred in the lattice model because
the tensile stresses in web exceeded the tensile failure stress criteria. Hence, it showed that the concrete member failed by tension, and not by compression. This mode of failure was rather similar to the splitting failure. The same modes of failure also occurred in the experiment, which verified the modes of failure obtained from the lattice model.

**Load-Displacement Response**

The following model behaviour were discussed and compared with the experimental behaviour:

- It was shown from the model (Figure 3a and 4a), that the mid-span deflection was not significantly affected by the formation of early flexural cracks. Thus, the slopes of the load versus deflection curves remained relatively unchanged after early flexural cracking. It was indicated as well in the experiment that the flexural cracking did not affect the stiffness or deflection of deep beams.

- It was observed in the experiment, that the load-deflection curves for deep beams were determined mainly by the shear deformation, which is a function of shear stiffness. Consequently, diagonal cracks significantly changed the slope of the curve because diagonal cracks reduce the shear stiffness. This was confirmed in the model since the load-deflection curves did change its slope after the formation of diagonal cracks (Figure 3c and 4c).

- It was observed in the model, that the following a change of slope of the load-deflection curves after the formation of a diagonal crack, the slope of the curve remained relatively unchanged afterwards (Figure 3e and 4e). This was attributed to the resistance provided by the “strut and tie” mechanism. The mechanism helped to maintain the apparent shear stiffness of the beam. As a result, even though the diagonal cracks reduced the shear stiffness, the overall slope did not change significantly. The same mechanism was observed in the experiment.

- The load-deflection curve also shows the way the beam has failed. It is shown in Figure 3g and 4g that the beam has failed in a brittle manner for both models, as were the case for the experimental beams. The brittle failures can be seen from the sudden drop in the load carrying capacity of the beams after the peak loads. The accuracy on the predicted load is shown in Figure 6 and Table 1.

**Conclusion**

The proposed lattice model has been shown to be suitable for modelling concrete deep beams and it can describe deep beam behaviour accurately. The load deflection curves show that the model is accurate for low load level up to the high ultimate load levels.

**References**


